McCreary et al (2001). Fundamental 2½-layer dynamics (Following LPS and Luyten and Stommel 1986)

The fundamental dynamics are that the Sverdrup flow is established quickly with mode 1 Rossby waves, then layer 2 Rossby characteristics are bent according to the Sverdrup velocities. Layer 2 characteristics are isolines of h ($= h_1 + h_2$), thus determine the shape of h_2 and flow in layer 2.

The dynamics are a steady, inviscid 2½-layer system. The momentum equations are linear but the continuity equations are not. Communication between layers is through Ekman upwelling, which occurs (upward only) when positive $Curl(\tau/f)$ ($=w_{Ek}$) raises h_1 up to a critical depth, where the upwelling can reach through the thermocline into the lower layer. (Note that there must be downwelling or a lower layer source/upper layer sink somewhere in the system to conserve mass, e.g. the circulation around Australia). Flow in layer 1 is Ekman plus geostrophic but flow in layer 2 is geostrophic only. Equations of motion are:

$$-fv_1 + p_{1x} = \tau^x/h_1\,, \qquad fu_1 + p_{1y} = \tau^y/h_1\,, \qquad (h_1u_1)_x + (h_1v_1)_y = w\,, \qquad p_1 = g'_{12}h_1 + g'_{23}h$$

$$-fv_2 + p_{2x} = 0\,, \qquad fu_2 + p_{2y} = 0\,, \qquad (h_2u_2)_x + (h_2v_2)_y = -w, \qquad p_2 = g'_{23}h$$

Imagine a wind stress pattern that has upwelling $Curl(\tau/f)$ in the far east, then weak downwelling w_{Ek} across the rest of the basin. $Curl(\tau)$ is positive across the basin in this band. This is roughly the situation along 7°-12°N in the Pacific (under the ITCZ). The Sverdrup flow ($\psi=(1/\beta)\int Curl(\tau)dx$) is a cyclonic gyre with poleward flow against the eastern boundary and weak equatorward geostrophic flow to the west of the upwelling region (any residual flow is taken up in the western boundary). Note that $hv = (1/\beta)Curl(\tau)$ but $hv_g = (f/\beta)w_{Ek}$. The eastern upwelling lifts h_1 in the east to the critical depth, and then transmits the upwelling through to layer 2, which also slopes upward to the west under the positive w_{Ek}. With linear continuity equations, Rossby characteristics are due west, and the layer 2 flow would be a cyclonic gyre with only zonal currents west of the upwelling region, feeding an equatorward western boundary current. Both h_1 and h_2 would be an east-west ridge. h_1 would slope gradually downwards to the west under the negative w_{Ek}, with consequent equatorward v_{g1} in the interior, while h_2 would be zonally flat west of the upwelling region ($v_{g2} = 0$) (until the two limbs connected in an equatorward western boundary current). Because f is smaller on the south side of the gyre, there would be more eastward than westward layer 2 flow, which would feed the upwelling at the east.

The nonlinearity leads to an equation for h: $v_gh_y + (u_g-c_r)h_x = 0$, where (u_g,v_g) are the depth-averaged geostrophic currents in the two layers, and c_r is the layer 2 Rossby wave speed. h is constant along the characteristics (u_g-c_r,v_g), and can be found by integrating along them from the west edge of the upwelling region (where h_1 and h_2 are known).

Characteristics emanate from the west edge of the upwelling region (as well as from the eastern boundary). In contrast to their purely zonal direction in the linear case, they are bent equatorward by v_g. In addition, where these characteristics traverse the region of positive u_g (the equatorward side of the Sverdrup gyre), their zonal component (u_g-c_r) can become zero or even positive. Thus there can be a significant zonal tilt to h_2 and interior meridional flow in layer 2. For the winds described above, h_2 will be tilted upwards to the west in the central Pacific and downwards in the far west, unlike the linear case where h_2 is zonally flat west of the upwelling region.